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# A note on brane inflation

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## Abstract

We demonstrate that there exists an inflationary solution on the positive tension brane in the Randall-Sundrum scenario. Inflation is driven by a slow-rolling scalar field on the brane and is achieved within the perturbative limit of the radion field. We find that inflation on the positive tension brane results in a slight increase in the separation between the two branes. However, we show that the slow-roll inflation is not possible on the negative tension brane. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Recently, an alternative mechanism of solving the hierarchy problem has been proposed [1]. The novel proposal exploits the fact that the fundamental Planck scale  $M$  is a TeV scale but in  $4 + d$  dimensions, where  $d$  is the number of extra dimensions which are to be compactified. The Planck scale in the observable world is generated because of the presence of large extra dimensions:  $M_{\text{Pl}}^2 = M^{2+d} V_d$ . Here  $V_d$  is the volume of the extra compactified dimensions. The proposal also demands that the standard model fields are bound to live in the observable world and gravity is the only force which mediates through the bulk and the observable world. The introduction of very large size of the compactified dimensions brings a completely new set of exciting

problems ranging from phenomenology of accelerator physics [2] to cosmology [3].

However, Randall and Sundrum [4] introduced another twist in the physics beyond 4 dimensions to account for the hierarchy between the Planck scale and the electroweak scale. They demonstrate that in a background of special non-factorizable geometry an exponential warp factor appears for the Poincaré invariant  $3 + 1$  dimensions. The model consists of two 3 branes situated at the fixed positions along the 5th dimension compactified on a  $S^1/Z_2$  orbifold symmetry. The space-time in the bulk is 5 dimensional anti de-Sitter space (AdS) (see also previous relevant work and more recent generalization [5]). The five dimensional Einstein's equations permit a solution which preserves 4 dimensional Poincaré invariance on the brane, with the metric taking the form

$$ds^2 = e^{-2k|z|} g_{\mu\nu} dx^\mu dx^\nu + dz^2. \quad (1)$$

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Here  $z$  is the extra spatial dimension and  $\mu, \nu = 0, \dots, 3$ . The constant  $k$  is determined by the bulk cosmological constant  $-\Lambda = 3M_5^3 k^2$  where  $M_5$  is the five dimensional Planck scale (see the 5 dimensional action defined later). The 4 dimensional Poincaré invariance also requires a fine-tuning of the brane tensions. Namely, the positive tension  $\sigma$  of the brane at  $z = 0$  is related to  $\Lambda$  and  $M_5$ , while the brane at  $z = z_c$  has equal and opposite tension  $-\sigma$ . Gravity is localized due to the exponentially decaying warp factor in the 5th dimension. The hierarchy between the Planck scale and the electroweak scale is explained by the suppression factor  $e^{-kz_c}$  on the negative tension brane, where the standard model (SM) particles are assumed to be.

The excitations around the background metric includes massless 4 dimensional graviton zero modes and a set of massive Kaluza-Klein modes which have masses proportional to  $1/z_c$ . In addition, there is a massless four-dimensional scalar associated with the relative motions of the two branes, i.e., the radion field [6]. The radion field would become massive once the separation of the two branes are stabilized due to a certain mechanism [7,8]. It is interesting to note that the reduced action of the massless graviton zero mode and the radion field in 4 dimensions on the branes, is not exactly the same as general relativity (GR). Instead, the effective four-dimensional action on the branes mimics that of a deviant theory of gravity, popularly known as scalar tensor theory, with the radion field taking the role of a Brans-Dicke scalar with non-trivial Brans-Dicke parameter [10]. In this theory, Einstein's gravity is recovered in a limiting case when the brane separation is stabilized and the radion field acquires mass. The cosmology of the Randall-Sundrum scenario have been under extensive study [9,12].

The purpose of this paper is to investigate the possibility of inflationary scenarios on the branes. We assume that the inflation is induced by a scalar field confined on the brane which has a slow-roll potential. In our discussion, we also assume that the inflation on the brane takes place before the stabilization of the extra dimensions [11], hence during the inflationary era, the radion field is massless and has no additional potential. We find that it is possible to have an inflationary era induced by a slow-rolling inflaton on the positive tension brane. In particular,

during this era the separation between the two branes increases. Namely, the bulk gravity reacts to inflation on the Planck brane and pushes the other brane away and enlarges the size of the AdS space. We find that the change in the size of the extra dimension can be sufficiently small such that the perturbative description of our framework is still valid towards the end of inflation. However, we fail to find a consistent inflationary solution within our assumption of a massless radion if we assume that a slow-rolling inflaton is confined on the negative tensioned brane.

In Section 2, we write down the effective actions on the branes. We present the equations of motions in the next section and the inflationary solutions on both the positive and the negative tension branes. In Section 4, we discuss our results.

## 2. Effective actions

The full action consists of 5 dimensional Einstein's gravity with a negative cosmological constant in the bulk. The two branes are located on the orbifold  $S^1/Z_2$  along the extra dimension, with the positive tension brane at  $z = 0$  and the negative tension brane at  $z = z_c$ .

$$S = 2 \int d^4x \int_0^{z_c} dz \sqrt{-g_5} \left[ \frac{M_5^3}{2} R_5 - 2\Lambda \right] - \sigma_+ \int d^4x \sqrt{-g_+} - \sigma_- \int d^4x \sqrt{-g_-}, \quad (2)$$

where  $M$  is the 5 dimensional Planck mass,  $\sigma_{\pm}$  and  $g_{\pm}$  are the brane tensions and the corresponding induced metric on the respective branes. The effective 4 dimensional Planck scale:  $M_{\text{Pl}} = M_5^3(1 - e^{-2kz_c})/(4k)$  [4]. The linearized gravity around its background including the massless degree of freedom, i.e., the massless 4 dimensional graviton and radion field, can be parametrized by the following metric solution [6],

$$ds^2 = e^{-2kh(x,z)} \bar{g}_{\mu\nu} dx^\mu dx^\nu + h_{,z}^2 dz^2, \quad h(x, z) = z + f(x) e^{2kz}. \quad (3)$$

where  $0 < z < z_c$ , and  $h_{,z}$  denotes the derivative with respect to  $z$ . The radion field  $f(x)$ , which is a

function of the brane coordinates only, was also introduced in [12]. For the classical solution, the induced metric on the positive tension brane at  $z = 0$  takes the form,

$$g_{+\mu\nu} = e^{-2kf} \bar{g}_{\mu\nu}, \quad (4)$$

and on the negative tension brane at  $z = z_c$ :

$$g_{-\mu\nu} = \alpha^{-1} e^{-2\alpha kf} \bar{g}_{\mu\nu}, \quad (5)$$

where  $\alpha \equiv e^{2kz_c}$  is a constant. With this metric ansatz one can integrate out the fifth dimension to achieve the effective four dimensional action, which mimics a scalar-tensor theory [14],

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi} \left[ \Phi R - \frac{\omega(\Phi)}{\Phi} (\nabla\Phi)^2 \right] + S_{m,\pm}, \quad (6)$$

with a varying scalar field  $\Phi$ , known as Brans-Dicke field and its coupling to gravity is determined by varying strength  $\omega$ .  $S_{m,\pm}$  is the action of matter fields that are confined on positive/negative tension brane. Note that the Brans-Dicke field  $\Phi$  and the coupling parameter  $\omega$  takes different values at positive and negative tension brane with our definition of the four dimensional metric on two branes, Eqs. (4), (5). The explicit definitions of the fields  $\Phi$  and the parameter  $\omega$  (dimensionful) in two coordinate frames are the following [10]

$$\Phi^\pm = \frac{2}{kG_5} e^{\mp k[(\alpha-1)f+z_c]} \sinh(k[(\alpha-1)f+z_c]), \quad (7)$$

$$\omega_\pm(f) = \pm 3e^{\pm k[(\alpha-1)f+z_c]} \sinh(k[(\alpha-1)f+z_c]) \times \left( 1 \mp \frac{e^{\pm k[(\alpha-1)f]} \sinh[k(\alpha-1)f]}{\sinh^2(kz_c)} \right), \quad (8)$$

where  $G_5$  is defined as  $G_5 \equiv (8\pi M_5^3)^{-1}$ . (For simplicity, we use the notation of  $\Phi^\pm$  and  $\omega_\pm$  in the following discussion where we study the inflationary scenario induced by scalar fields on positive/negative tension brane separately.) The effective action Eq. (6) is the correct description of the dynamics of the massless fields as long as the perturbative limit

$e^{2k\alpha f} \ll \alpha$  is satisfied<sup>1</sup>. It is pointed out in [10] that  $\Phi$  is the scalar Brans-Dicke field, whose dynamical evolution determines the strength of gravity, namely, the Newton's constant  $G_N \equiv 1/\Phi$ . The strength of the coupling between gravity and the scalar field  $\Phi^\pm$  is determined by  $\omega_\pm$ . When  $\omega_\pm \rightarrow \infty$ , the scalar tensor theory approaches Einstein's general theory of relativity provided  $\omega'_\pm/\omega_\pm^3 \rightarrow 0$  [15]. The present observations such as light bending, perihelion precession and radar echo delay phenomena in the solar system suggests  $\omega_\pm > 3000$  [16]. In the perturbative limit  $\omega_+$  on the positive tension brane  $\omega_+ \sim 3\alpha e^{2k\alpha f}/2$  easily satisfies the experimental constraint, while on the negative tension brane  $\omega_- \sim -3/2$  does not satisfy this bound [10]. However, we take the view that the situation could be altered if additional massive degrees of freedom in the bulk associated with stabilization are included. In the following section we investigate the dynamics of the branes in the presence of a scalar field inflaton with a slow-roll potential on the brane. We study the inflationary aspects of the solutions and the evolution of the fields  $\Phi^\pm$  and the parameters  $\omega_\pm$ .

### 3. Inflationary solutions

In this section we study the feasibility of inducing inflation by adding a matter Lagrangian on positive and negative tension brane, respectively. We conjecture that adding an additional scalar field on the branes will not affect the background classical solution of the gravity and the dynamics can be safely described with the usual perturbative theory [6]. At the end we shall see that inflation initiated on the positive tension brane indeed satisfies our assumption, while an inconsistency appears for the negative tension brane. The action of a scalar field  $\chi$  on the branes takes the following form,

$$S_{m,\pm} = \int d^4x \sqrt{-g_\pm} [g_{\pm\mu\nu} \partial^\mu \chi \partial^\nu \chi - V(\chi)], \quad (9)$$

<sup>1</sup> Here we want to emphasise that in the strict limit  $e^{2k\alpha f} \ll \alpha$ , the effective action on the brane (6) is equivalent to the original action in five dimensions. It is possible that the effective brane action deduced from dimensional reduction could differ in higher orders from that of the original 5-dimensional action [13].

where  $g_{\pm\mu\nu}$  are the induced metric on the branes as defined previously.

To discuss inflation on the brane, we choose  $\bar{g}_{\mu\nu}$  to be the Friedmann-Robertson-Walker (FRW) metric,  $\bar{g}_{\mu\nu} = \text{diag}\{-1, A(t), A(t), A(t)\}$ , where  $A(t)$  is the scale factor. In order to derive the equations of motions for the fields, one can perform a coordinate transformation on the brane,  $d\tau = \sqrt{|g_{\pm 00}|} dt$ , such that the metric on the branes, Eqs. (4), (5) can be brought to the ordinary FRW metric by defining the scale factors  $a_+(\tau) \equiv e^{-kf(t)}A(t)$  and  $a_-(\tau) \equiv \alpha^{-1/2}e^{-\alpha kf(t)}A(t)$ . We have explicitly assumed that the radion field  $f$  has solely the time dependence. Since we are interested in studying the inflationary solution, we can appropriately assume that the late time dynamics of  $f$  does not depend on the spatial coordinates of the branes. Under these assumptions the equations for the scalar fields [17] are

$$H^2 + H \frac{\dot{\Phi}^\pm}{\Phi^\pm} - \frac{\omega}{6} \frac{\dot{\Phi}^{\pm 2}}{\Phi^{\pm 2}} = \frac{8\pi}{3\Phi^\pm} \left( \frac{1}{2} \dot{\chi}^2 + V(\chi) \right), \quad (10)$$

$$\ddot{\Phi}^\pm + \dot{\Phi}^\pm \left[ 3H + \frac{\dot{\omega}}{2\omega + 3} \right] = 8\pi \frac{4V(\chi) - \dot{\chi}^2}{2\omega + 3}, \quad (11)$$

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = 0, \quad (12)$$

where the Hubble parameter  $H \equiv \dot{a}_\pm/a_\pm$ , the over-dot denotes  $d/d\tau$  and the prime denotes  $d/d\chi$ . An inflationary epoch, in which the scale factors  $a_\pm$  are accelerating, requires the scalar field  $\chi$  to evolve slowly compared to the expansion of the Universe. Thus, the following conditions of slow-rolling are required:

$$\ddot{\chi} \ll H\dot{\chi}, \quad (13)$$

$$\frac{1}{2} \dot{\chi}^2 \ll V(\chi), \quad (14)$$

$$\ddot{\Phi}^\pm \ll H\dot{\Phi}^\pm \ll H^2\Phi^\pm. \quad (15)$$

Under the slow-roll conditions, the Eqs. (10)–(12) can be simplified. In the following subsections, we discuss the possible solutions on the positive and the negative tension branes respectively. For simplicity, we assume that  $V(\chi) = V_0$  during the epoch when the slow-roll conditions Eqs. (13)–(15) are satisfied. There are other forms of  $V(\chi)$  that one can take to

satisfy the slow-roll conditions [14], which we will not consider in this paper.

### 3.1. On the positive tension brane

On the positive tension brane, when  $e^{2k\alpha f} \ll \alpha$ ,  $\Phi^+ \approx G_p(1 - e^{(-2k\alpha f)}/\alpha)$ , and the Brans-Dicke parameter  $\omega_+$  can be written as a function of  $\Phi^+$ ,

$$\omega_+(\Phi^+) \approx \frac{3}{2} \frac{\Phi^+}{G_p - \Phi^+}, \quad (16)$$

where  $G_p \equiv 1/(kG_5)$ . In this limit, the value of  $\Phi^+$  approaches  $G_p$ . And  $\omega_+ \sim \frac{3}{2}\alpha \gg 1$ . Following the slow-roll approximations Eqs. (13)–(15), Eqs. (10), (11) can be expressed as:

$$3H^2(G_p - \Phi^+) \approx 8\pi V_0(\Phi^+)^{-1}(G_p - \Phi^+) + \frac{3}{4}(\dot{\Phi}^+)^2(\Phi^+)^{-1}, \quad (17)$$

$$3H\dot{\Phi}^+(G_p - \Phi^+) + \frac{1}{2}\dot{\Phi}^{+2}(\Phi^+)^{-1}G_p \approx \frac{32}{3}\pi V_0 \frac{(G_p - \Phi^+)^2}{\Phi^+}. \quad (18)$$

By manipulating Eqs. (17), (18) we get:

$$\frac{(\dot{\Phi}^+)^2}{(\Phi^+)} \frac{G_p}{G_p - \Phi^+} \approx 6H \left[ \frac{4}{3}H(G_p - \Phi^+) - \dot{\Phi}^+ \right], \quad (19)$$

where the slow-roll condition Eq. (15) has been used to neglect the term  $\frac{3}{4}(\dot{\Phi}^+)^2(\Phi^+)^{-1}$ . The above equation can be solved approximately by:

$$\frac{\dot{\Phi}^+}{G_p - \Phi^+} \approx \gamma H, \quad (20)$$

where  $\gamma = \sqrt{17} - 3$ , we have used Eq. (15) and the approximation that in the limit  $e^{2\alpha k f} \ll \alpha$ ,  $\Phi^+ \sim G_p$ . With Eq. (20) and the slow-roll condition, Eq. (17) can be reduced to

$$H \approx \sqrt{\frac{8\pi V_0}{3\Phi^+}}. \quad (21)$$

One can then solve for  $\Phi^+$  from Eq. (20),

$$(\tau - \tau_0) \sqrt{\frac{2\pi\gamma^2}{3} \pi V_0} = \left[ -\sqrt{\Phi^+} + \frac{\sqrt{G_p}}{2} \ln \frac{\sqrt{G_p} + \sqrt{\Phi^+}}{\sqrt{G_p} - \sqrt{\Phi^+}} \right]_{\Phi_0^+}^{\Phi^+}, \quad (22)$$

where  $\Phi_0^+$  is the initial value of  $\Phi^+$ . We take as an initial condition that at the beginning of the inflation  $f(\tau = \tau_0) = 0$ , hence  $\Phi_0^+ = G_p(1 - 1/\alpha)$ . Solving Eq. (20) for the scale factor, we get:

$$\frac{a(\tau)}{a_0} \simeq \left[ \frac{G_p - \Phi_0}{G_p - \Phi(t)} \right]^{1/\gamma}. \quad (23)$$

Substituting Eq. (23) in Eq. (22) and assuming  $a(\tau)/a_0 \gg 1$  at  $\tau \gg \tau_0$ , we get the final expression

$$\frac{a(\tau)}{a_0} \simeq \exp \left( \sqrt{\frac{8\pi V_0}{3G_p}} (\tau - \tau_0) \right). \quad (24)$$

The above equation confirms the exponential growth in the scale factor during the inflationary era. The evolution of the Brans-Dicke field can be obtained by substituting Eq. (24) into Eq. (22).

$$\frac{\Phi^+(\tau)}{G_p} \approx 1 - \frac{1}{\alpha} \exp \left( -2 \left[ \sqrt{\frac{2\pi\gamma^2 V_0}{3G_p}} (\tau - \tau_0) \right] \right), \quad (25)$$

where we have used the initial condition  $f(\tau = \tau_0) = 0$  and  $\Phi_0^+ = G_p(1 - 1/\alpha)$ . Eq. (25) shows that the effective Brans-Dicke field grows with time during the inflationary era and approaches its asymptotic value,  $(kG_5)^{-1}$  at  $\tau \rightarrow \infty$ . We can also estimate the final value of  $f(\tau_f)$  in terms of the number of e-foldings  $N$  of the inflation, by using the expression for  $\Phi_+$ . From Eq. (24) and (25), we get:

$$f(\tau_f) = \frac{\gamma N}{2k\alpha}, \quad (26)$$

where,

$$N \equiv \ln \left( \frac{a(\tau_f)}{a(\tau_0)} \right). \quad (27)$$

If we assume that  $N \sim 60$  e-foldings (to solve the flatness problem in the usual hot big bang Universe),

it is not hard to realize that for sufficiently large  $\alpha$ , the condition  $e^{2k\alpha f(\tau_f)} \sim e^{\gamma N} \ll \alpha$  could be satisfied such that during the complete process of inflation, the perturbative description of the radion field remains valid. However, note that the size of  $\alpha$  that is required here is larger than the required size of  $\alpha$  to explain the hierarchy problem, i.e.,  $e^{kz_c} \sim 10^{15}$  is needed to produce TeV scale masses from the fundamental scale of  $M_{\text{pl}} \sim 10^{19}$  GeV [4]. Since, we have assumed that the stabilization of the brane separation and the inflation are two independent processes taking place at different times, it is possible that the stabilization mechanism happens after the end of inflation and reduces the separation of the two branes. However, if  $z_c$  at the onset of inflation does not satisfy  $e^{\gamma N} \ll \alpha$ , inflation would eventually destabilize the configuration of the two branes and the perturbative approach would break down. We also make no comments on how the inflation is stopped, however, we believe that the end of slow-roll conditions eventually leads to graceful exit of inflation. Though it would be interesting to investigate the possibility of the stabilization mechanism as a way to stop inflation and generate reheating, it is beyond the scope of this work. A similar expression can be derived for  $\Phi^+(\tau_f)$  in terms of the number of e-foldings  $N$ ,

$$\frac{\Phi^+(\tau_f)}{G_p} = 1 - \frac{e^{-\gamma N}}{\alpha}. \quad (28)$$

In the perturbative limit,  $f$ , the radion field determines the perturbation of the separation between the two branes. The result in Eq. (26) shows, that by assuming  $f = 0$  as an initial condition for inflation to occur,  $f(\tau)$  increases during inflation. The increased value depends on the number of e-foldings that can be achieved during inflation. Therefore, inflation on the Planck brane results in increasing the separation between the two branes, i.e., the size of the AdS space. On the other hand Eq. (28) suggests that the increase in the radion field  $f(\tau)$  also leads to gradual increase in  $\Phi^+$  and at the end of inflation its value asymptotically approaches  $G_p \sim 1/(kG_5)$ .

### 3.2. On the negative tension brane

In this section we study the feasibility of an inflationary solution initiated by scalar fields on the

negative tension brane. The action on the negative tension brane is given in Eq. (6) and Eq. (9) with negative superscript. Again in the perturbative limit,  $\Phi^-$  in Eq. (7), reduces to a simple form

$$\Phi^- \approx \frac{1}{kG_5} \alpha e^{2k(\alpha-1)f}, \quad (29)$$

and the parameter:

$$\omega_- \approx -\frac{3}{2} \left( 1 + \frac{2}{\alpha} - \frac{3}{kG_5\Phi^-} \right). \quad (30)$$

Within the perturbative limit  $e^{2k\alpha f} \ll \alpha$ ,  $2\omega_- + 3$  is greater than zero, provided

$$f(\tau) < \frac{\ln(3/2)}{2k\alpha}. \quad (31)$$

In the perturbative limit, we also have:

$$kG_5\Phi^- \approx \alpha \gg 1, \quad (32)$$

for  $f \sim 0$ . Note, that the situation is exactly opposite to that of the positive tension brane, where  $\Phi^+$  is small and  $\omega_+ \gg 1$ .

With these assumptions and with the help of slow-roll conditions Eqs. (13)–(15), Eq. (10) can be simplified to yield,

$$3H^2 \approx \frac{8\pi V_0}{\Phi^-}. \quad (33)$$

Here we have used Eq. (30), and we have also assumed the initial condition  $f(\tau = \tau_0) \sim 0$ . On the other hand Eq. (11), around  $f \sim 0$ , reduces to:

$$\frac{\dot{\Phi}^-}{\Phi^-} \approx \frac{16\pi}{27} \frac{kG_5 V_0}{H(1 - G_\phi \Phi^-)}, \quad (34)$$

where  $G_\phi \equiv \frac{2}{3} \frac{kG_5}{\alpha}$ , such that around  $f \sim 0$ ,  $G_\phi \Phi^- \sim 2/3$ . Hence, the slow-roll condition:  $\dot{\Phi}^-/\Phi^- \ll H$  requires  $kG_5 V_0/H \ll H$ . However, it is easy to show that combined with the result of Eq. (33) this leads to an inconsistent result  $kG_5 \Phi^- \ll 1$ , compared to Eq. (32). It leads to the conclusion that within the perturbative limit the slow-roll inflation on the negative tension brane breaks down from the very beginning.

The result can be understood as follows. On the negative tension brane, the effective physical scale is reduced by a factor of  $\alpha \equiv e^{2kz_c}$  compared to the fundamental scale  $M_{\text{pl}}$ , the same mechanism used by Randall-Sundrum to explain the hierarchy between

the Planck scale and the electroweak scale [4]. Hence, inflation is driven by the flat potential  $V_0/\alpha$  of  $\chi$  Eq. (33). On the other hand, due to the fact that  $\omega_-$  is small on the negative brane, in fact  $2\omega_- + 3 \sim 1/\alpha$ , the coupling between  $\Phi^-$  and the inflaton  $\chi$  is boosted by a factor of  $\alpha$  (see the term on the right-hand side of Eq. (11)). This creates an effective potential for  $\Phi^-$  with a steep slope, hence the evolution of  $\Phi^-$  is no more subdominant compared to the Hubble expansion rate which is set here by the reduced strength of  $V_0$ . And eventually the slow-roll assumptions break down. On the other hand, one may also conclude that the inflation quickly pushes the two branes out of their equilibrium positions, hence, destabilizes the configuration.

Note that the inconsistency is related to the non-physical value of the coupling between gravity and the Brans-Dicke field  $\Phi^-$  on the negative tension brane,  $\omega_-$ . One may consider a situation where the stabilization process is inter-winded with the inflation. By including new contributions from massive particles in the bulk, the effective steep potential of  $\Phi^-$  could be balanced to provide a possible solution to the problem. (It was suggested by some of the studies on cosmology in the Randall-Sundrum scenario [12].) Inflation in such a case would be driven by the massive Kaluza-Klein modes of the bulk field, which could possibly assist inflation [18] on the brane. Work in this direction is in progress.

#### 4. Discussion

The low energy effective four dimensional action of gravity in the Randall-Sundrum scenario mimics that of a scalar tensor theory with a varying Brans-Dicke parameter before the brane separation is stabilized. Assuming inflation takes place before the stabilization of the brane separation, we have shown that it is possible to discuss perturbatively the dynamics of inflation with an additional matter Lagrangian on the positive tension brane. We find that the slow-rolling inflaton on the positive tension brane induces inflation, during which the radion field is displaced. The displacement can be sufficiently small that the perturbative description is still valid. During the expansion of the brane, the separation between

the two branes increases slightly and this could be the initial condition for a subsequent process in which the brane separation is stabilized and the radion field becomes massive. However, on the negative tension brane, the slow-roll inflation destabilizes the brane separation at the very beginning, and it is not possible to reach the general relativity limit for a given separation between the two branes.

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## References

- [1] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* 246 (1990) 377; *Phys. Rev. D* 60 (1990) 104002; I. Antoniadis, S. Dimopoulos, G. Dvali, *Nucl. Phys. B* 516 (1999) 70; N. Arkani-Hamed, S. Dimopoulos, J. March-Russell, *hep-th/9809124*.
- [2] L.J. Hall, D. Smith, *Phys. Rev. D* 60 (1999) 085008; T.G. Rizzo, *Phys. Rev. D* 59 (1999) 115010; T. Han, J.D. Lykken, R.-J. Zhang, *Phys. Rev. D* 59 (1999) 105006; G.F. Giudice, R. Rattazi, J.D. Wells, *Nucl. Phys. B* 544 (1999) 3; E.A. Mirabelli, M. Perelstein, M.E. Peskin, *Phys. Rev. Lett.* 82 (1999) 2236; J.L. Hewett, *Phys. Rev. Lett.* 82 (1999) 4765.
- [3] D.H. Lyth, *Phys. Lett. B* 448 (1999) 191; N. Kaloper, A. Linde, *Phys. Rev. D* 59 (1999) 101303; A. Mazumdar, *Phys. Lett. B* 469 (1999) 55; T. Nihei, *Phys. Lett. B* 465 (1999) 81; E. Halyo, *Phys. Lett. B* 561 (1999) 109; R. Maartens, D. Wands, B.A. Bassett, I. Heard, *hep-ph/9912464*; N. Kalpoer, A.R. Liddle, *Phys. Rev. D* 61 (2000) 123513.
- [4] L. Randall, R. Sundrum *Phys. Rev. Lett.* 83 (1999) 3370; *Phys. Rev. Lett.* 83 (1999) 4690.
- [5] M. Cvetič, S. Griffies, S. Rey, *Nucl. Phys.* 381 (1992) 301; M. Cvetič, H.H. Soleng, *Phys. Rep.* 282 (1997) 159; M. Cvetič, J. Wang, *Phys. Rev. D* 61 (2000) 124020.
- [6] C. Charmousis, R. Gregory, V.A. Rubakov, *hep-th/9912160*.
- [7] W. Goldberger, M. Wise, *hep-ph/9911457*; *Phys. Rev. Lett.* 83 (1999) 4922.
- [8] O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, *hep-th/9909134*; S.S. Gubser, *hep-th/9912001*.
- [9] P. Binetruy, C. Deffayet, D. Langlois, *Nucl. Phys. B* 565 (2000) 269; N. Kaloper, *Phys. Rev. D* 60 (1999) 123506.
- [10] T. Chiba, *Phys. Rev. D* 62 (2000) 021502.
- [11] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, J. March-Russell, *hep-ph/9903239*; *hep-ph/9903224*.
- [12] G. Dvali, S.H.H. Tye, *Phys. Lett. B* 450 (1999) 72; C. Csaki, M. Graesser, J. Terning, *Phys. Lett. B* 456 (1999); C. Csaki, M. Graesser, L. Randall, J. Terning, *hep-ph/9911406*; J.M. Cline, *Phys. Rev. D* 61 (2000) 023513; E.E. Flanagan, S.H.H. Tye, I. Wasser, *hep-ph/9909373*; A. Riotto, *hep-ph/9904485*; H.B. Kim, H.D. Kim, *Phys. Rev. D* 61 (2000) 064003; R.N. Mohapatra, A. Pérez-Lorenzana, C.A. de S. Pires, *hep-ph/0003328*.
- [13] T. Shiromizu, K. Maeda, M. Sasaki, *gr-qc/9910076*.
- [14] C. Brans, R.H. Dicke, *Phys. Rev.* 124 (1961) 925; P.G. Bergmann, *Int. J. Theor. Phys.* 1 (1968) 25; R.V. Wagoner, *Phys. Rev. D* 1 (1970) 3209; C.M. Will, *Theory and experiment in gravitational physics*, Cambridge University Press, Cambridge, 1993.
- [15] K. Nordvedt, *Astrophys. J* 161 (1970) 1059.
- [16] C.M. Will, *gr-qc/9811036*.
- [17] J.D. Barrow, *Phys. Rev. D* 51 (1995) 2729.
- [18] A.R. Liddle, A. Mazumdar, F. Schunck, *Phys. Rev. D* 58 (1998) 061301; E.J. Copeland, A. Mazumdar, N.J. Nunes, *Phys. Rev. D* 60 (1999) 083506.